CHAPTER 6 (MEASUREMENT)

WORKED SOLUTIONS TO EXERCISES ON MEASUREMENT

SECTION I – MULTIPLE CHOICE

1 $367000 = 3.67 \times 10^5$

2 $8.5\text{m} = 8.5 \times 1000\text{mm}$
$= 8500\text{mm}$

3 Distance travelled $= (2.24 \times 2000)\text{m}$
$= 4480\text{m}$

4 $C = 2\pi r$
$= 2 \times \pi \times 6400$
$= 40212\text{km}$

5 Right $\angle d\Delta$

6 Hypotenuse

7 $h^2 = 7^2 + 8^2$
$h^2 = 49 + 64$
$h^2 = 113$
$h = \sqrt{113}$ m

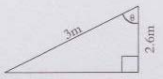
8 Similar

9 Equiangular

10 Same shape

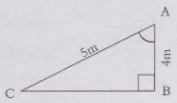
11 $12\sin 85^\circ \approx 12\ (0.996194698)$
≈ 11.95433638
$= 11.95$

12 $\cos\theta = \dfrac{2.6}{3}$



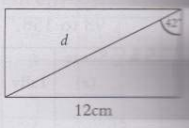
$\cos\theta \approx 0.866666...$
$\theta \approx 29^\circ\ 56'$
$\theta \approx 30^\circ$ (to nearest degree)

13 $\cos A = \dfrac{4}{5}$



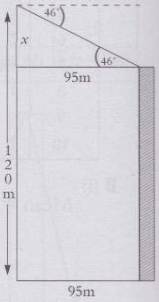
$\cos A \approx 0.8$
$A \approx 36^\circ\ 52'$
$A \approx 37^\circ$ (to nearest degree)

14 $\sin 42^\circ = \dfrac{12}{d}$



$d \sin 42^\circ = 12$
$d = \dfrac{12}{\sin 42^\circ}$
$d \approx 17.9337186$
$d \approx 17.9\text{cm}$ (1 d.p.)

15 $\dfrac{x}{95} = \tan 46^\circ$



$x = 95 \tan 46^\circ$
$x \approx 98.3753798$
$h = 120 - 98.3753798$
$h \approx 21.6246202\text{m}$
$h = 22\text{m}$ (to nearest metre)

16 Area $= s^2$
$A = 15^2$
$A = 225\text{cm}^2$

17 $1\text{m}^2 = 10\,000\text{cm}^2$

18 Volume of the cube $= 5^3 = 125\text{cm}^3$
Volume of a spherical ball $= \dfrac{4}{3}\pi r^3$
$= \dfrac{4}{3} \times \pi \times \left(\dfrac{0.5}{2}\right)^3$
≈ 0.065449846
Number of spherical balls
$= \dfrac{125}{0.065449846}$
≈ 1909.859317
$= 1900$

19 $V = \dfrac{1}{3}\pi r^2 h$
$= \dfrac{1}{3} \times \pi \times (3.5)^2 \times 8$
$= 102.62536$
$= 103\text{cm}^3$

$= 84m^3$

(ii) $1m^3 = 1000L$

$84m^3 = 84 \times 1000L$

$= 84000L$

B $23 \times 16^7 = 6174015488$

$\approx 6.174015488 \times 10^9$

$\approx 6.17 \times 10^9$ (3 s.f.)

C $7.68 \times 10^6 \div 312906$

≈ 24.54411229

$= 24.6$ (1 d.p.)

D (i) $A = \pi r^2 = \pi \times 6^2$

≈ 113.0973355

$\approx 113.1m^2$ (1 d.p.)

(ii) Area of two laneways $= \pi R^2 - \pi r^2$

$= \pi(13)^2 - \pi(6)^2$

$= 169\pi - 36\pi$

$= 133\pi$

≈ 417.8318229 (Cal.)

$\approx 417.83m^2$ (2 d.p.)

22 A (i) Δs are equiangular and hence similar

$\therefore$ their sides are in the same ratio

$\frac{x}{4} = \frac{9}{6}$ $\quad \therefore 6x = 36$

$x = 6$

(ii) Δs are equiangular and hence similar

$\therefore \frac{x}{1} = \frac{3}{2}$ $\quad \therefore x = \frac{3}{2}$

$x = 1.5$

$\angle PQR = \angle PST$ (Corresponding $\angle$s)

$\angle PRQ = \angle PTS$ (Corresponding $\angle$s)

$\therefore \Delta$s are equiangular and hence similar

D i) True ii) False iii) False

iv) True v) False

23 A (i) $S.A. = 4\pi r^2$

$S.A. = 4 \times \pi \times (6400)^2$

$= 514718540.4$ (Cal.)

$= 5.15 \times 10^8 km^2$ (3 s.f.)

(ii) $V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3} \times \pi \times (6400)^3$

$V = 1.098066219 \times 10^{12} km^3$

$V = 1.098 \times 10^{12} km^3$ (4 s.f.)

B diameter $= 10.6m$

radius $= 5.3m$

$A = \pi rs$

$= \pi \times 5.3 \times 7$

$= 116.5330874$

$= 116.55cm^2$ (2 d.p.)

C (i) diameter $= 18cm$

radius $= 9cm$

$V = \pi r^2 h$

$= \pi \times (9)^2 \times 12$

$= 3053.628059cm^3$

$= 3053.7cm^3$ (2 d.p.)

(ii) $1cm^3 = 1mL$

$\therefore$ Capacity $= 3053.7mL$

$= 3054mL$ to the nearest mL

24 A (i) S.A. $= 2(14 \times 12) + 2(12 \times 10) + 2(14 \times 10)\ \text{cm}^2$

S.A. $= 2(168) + 2(120) + 2(140)$

S.A. $= 336 + 240 + 280$

S.A. $= 856\text{cm}^2$

(ii) S.A. $= 2(\frac{1}{2} \times 10 \times 12) + (10 \times 25) + 2(25 \times 13)$

$= 2(60) + 250 + 2(325)$

$= (120 + 250 + 650)\text{cm}^2$

$= 1020\text{cm}^2$

(iii) S.A. $= 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 7^2 + 2 \times \pi \times 7 \times 25$

$= 98\pi + 350\pi$

$= 448\pi\text{cm}^2$

$= 1407.433509\text{cm}^2$ (Cal.)

$= 1407.4\text{cm}^2$ (1 d.p.)

B (i) $V = \frac{1}{3}Ah$

$= \frac{1}{3}(8.6)^2 \times 12$

$= 295.84\text{cm}^3$

(ii) $V = \pi r^2 h + \frac{1}{3}\pi r^2 H$

$V = \pi \times 12^2 \times 28 + \frac{1}{3} \times \pi \times 12^2 \times 16$

$V = 4032\pi + 768\pi\text{cm}^3$

$V = 4800\pi\text{cm}^3$

$V = 15079.6\text{cm}^3$

(iii) diameter = 36cm

radius = 18cm

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3} \times \pi \times 18\,\text{cm}^3$

$V = 7776\pi\text{cm}^3$

$V \approx 24429.02447\text{cm}^3$

$V \approx 24429\text{cm}^3$

25 A (i) $\sin 40° - \tan 30° + \cos 65°$

$= 0.642787609 - 0.577350269 + 0.422618261$

≈ 0.488055601

$= 0.49$ (2 d.p.)

(ii) $\frac{\sin 23° + \cos 23°}{\tan 23°}$

$\approx \frac{0.390731128 + 0.920504853}{0.424474816}$

$\approx \frac{1.311235982}{0.424474816}$

≈ 3.08907839

≈ 3.09 (2 d.p.)

B (i) $\cos\theta = \frac{3}{7}$

$\theta \approx 64.62306648$ (Cal.)

$\theta \approx 64° 37'$

(ii) $\sin\theta = \frac{15}{17}$

$\theta \approx 61.92751307$ (Cal.)

$\theta \approx 61° 56'$

(iii) $\tan\theta = 0.5896$

$\theta \approx 30.52360142$ (Cal.)

$\theta \approx 30° 31'$

C (i) $\frac{y}{18.67} = \sin 40°$

$\therefore y = 18.67 \sin 40° = 12\text{cm}$

(ii) $\frac{x}{8.7} = \cos 45°$

$x = 8.7 \cos 45°$

$x \approx 6.151828997$ (Cal.)

$x \approx 6.15$ (2 d.p.)

D (i) $\tan\theta = \frac{6.9}{8.65}$

$\theta \approx 0.797687861$

$\theta \approx 38° 35'$

6 A $P(E) = \frac{n(E)}{n(S)} = \frac{3}{9} = \frac{1}{3}$

B $P(E) = \frac{n(E)}{n(S)} = \frac{2}{9}$

C $P(E) = \frac{n(E)}{n(S)} = \frac{5}{9}$

7 $P(E) = \frac{n(E)}{n(S)} = \frac{7}{28} = \frac{1}{4}$

8 A $P(E) = \frac{n(E)}{n(S)} = \frac{5}{10} = \frac{1}{2}$

B $P(E) = \frac{n(E)}{n(S)} = \frac{4}{10} = \frac{2}{5}$

C $P(E) = \frac{n(E)}{n(S)} = \frac{5}{10} = \frac{1}{2}$

D $P(E) = \frac{n(E)}{n(S)} = \frac{2}{10} = \frac{1}{5}$

E $P(E) = \frac{n(E)}{n(S)} = \frac{0}{10} = 0$

F $P(E) = \frac{n(E)}{n(S)} = \frac{3}{10}$

9 $P(E) = \frac{n(E)}{n(S)} = \frac{6}{90} = \frac{1}{15}$

10 $\frac{1}{3} \times 90 = 30$ times

WORKED SOLUTIONS TO EXERCISES ON PROBABILITY

SECTION I – MULTIPLE CHOICE

1 $P(E) = \frac{n(E)}{n(S)} = \frac{13}{52} = \frac{1}{4}$

2 $P(E) = \frac{n(E)}{n(S)} = \frac{4}{9}$

3 $P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

4 $P(E) = \frac{n(E)}{n(S)} = \frac{28}{52} = \frac{7}{13}$

5 $P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

6 $P(E) = \frac{n(E)}{n(S)} = \frac{5}{14}$

7 $P(E) = \frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}$

8 $P(E) = \frac{n(E)}{n(S)} = \frac{2}{3}$

9 $P(E) = \frac{n(E)}{n(S)} = \frac{4}{11}$

10 $P(E) = \frac{n(E)}{n(S)} = \frac{1}{3}$

11 $P(E) = \frac{n(E)}{n(S)} = \frac{3}{10}$

12 $P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

13 $P(E) = \frac{n(E)}{n(S)} = \frac{1}{52}$

14 $P(E) = \frac{n(E)}{n(S)} = \frac{2}{9}$

15 $P(E) = \frac{n(E)}{n(S)} = \frac{5}{10} = \frac{1}{2}$

16 $P(E) = \frac{n(E)}{n(S)} = \frac{10}{25} = \frac{2}{5}$

17 $P(E) = \frac{n(E)}{n(S)} = \frac{8}{20} = \frac{2}{5}$

18 $P(E) = \frac{n(E)}{n(S)} = \frac{1}{3}$

19 $P(E) = \frac{n(E)}{n(S)} = \frac{1}{36}$

20 $P(E) = \frac{n(E)}{n(S)} = \frac{50}{1000} = \frac{1}{20}$

21 A (i) $P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$

(ii) $P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

(iii) $P(E) = \frac{n(E)}{n(S)} = \frac{0}{6} = 0$

(iv) $P(E) = \frac{n(E)}{n(S)} = \frac{0}{6} = 0$

(v) $P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

B $P(E) = \frac{n(E)}{n(S)} = \frac{2}{2} = 1$

C (i) $P(E) = \frac{n(E)}{n(S)} = \frac{13}{52} = \frac{1}{4}$

(ii) $P(E) = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

(iii) $P(E) = \frac{n(E)}{n(S)} = \frac{26}{52} = \frac{1}{2}$

(iv) $P(E) = \frac{n(E)}{n(S)} = \frac{8}{52} = \frac{2}{13}$

(v) $P(E) = \frac{n(E)}{n(S)} = \frac{39}{52} = \frac{3}{4}$

22 A (i) $P(E) = \frac{n(E)}{n(S)} = \frac{5}{9}$

(ii) $P(E) = \frac{n(E)}{n(S)} = \frac{2}{9}$

(iii) $P(E) = \frac{n(E)}{n(S)} = \frac{3}{9} = \frac{1}{3}$

(iv) $P(E) = \frac{n(E)}{n(S)} = \frac{4}{9}$

B (i) $P(E) = \frac{n(E)}{n(S)} = \frac{7}{20}$

(ii) $P(E) = \frac{n(E)}{n(S)} = \frac{5}{20} = \frac{1}{4}$

(iii) $P(E) = \frac{n(E)}{n(S)} = \frac{12}{20} = \frac{3}{5}$

(iv) $P(E) = \frac{n(E)}{n(S)} = \frac{0}{20} = 0$

23 A (i) $P(E) = \frac{n(E)}{n(S)} = \frac{20}{100} = \frac{1}{5}$

(ii) $P(E) = \frac{n(E)}{n(S)} = \frac{9}{100}$

(iii) $P(E) = \frac{n(E)}{n(S)} = \frac{28}{100} = \frac{7}{25}$

(iv) $P(E) = \frac{n(E)}{n(S)} = \frac{19}{100}$

(v) $P(E) = \frac{n(E)}{n(S)} = \frac{26}{100} = \frac{13}{50}$

(vi) $P(E) = \frac{n(E)}{n(S)} = \frac{10}{100} = \frac{1}{10}$

B (i) $P(E) = \frac{n(E)}{n(S)} = \frac{21}{26}$

(ii) $P(E) = \frac{n(E)}{n(S)} = \frac{5}{26}$

(iii) $P(E) = \frac{n(E)}{n(S)} = \frac{3}{26}$

(iv) $P(E) = \frac{n(E)}{n(S)} = \frac{2}{26} = \frac{1}{13}$

24 A (i) $P(E) = \frac{n(E)}{n(S)} = \frac{1}{4}$

(ii) $P(E) = \frac{n(E)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

(iii) $P(E) = \frac{n(E)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

(iv) $P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}$

(v) $P(E) = \frac{n(E)}{n(S)} = \frac{0}{4} = 0$

B (i) $P(E) = \frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

(ii) $P(E) = \frac{n(E)}{n(S)} = \frac{9}{36} = \frac{1}{4}$

(iii) $P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(iv) $P(E) = \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

25 A $\frac{13}{50}$

B $\frac{13}{50}$

C yes

D $P(E) = \frac{n(E)}{n(S)} = \frac{28}{50} = \frac{14}{25}$

E $P(E) = \frac{n(E)}{n(S)} = \frac{5}{50} = \frac{1}{10}$

26 A $\frac{4}{30} = \frac{2}{15}$

B $\frac{2}{15}$

C yes

D $P(E) = \frac{n(E)}{n(S)} = \frac{10}{30} = \frac{1}{3}$

E $P(E) = \frac{n(E)}{n(S)} = \frac{1}{30}$

$$\frac{\sin x}{10} = \frac{\sin 36}{16}$$

$$\sin x = \frac{10 \sin 36}{16}$$

$$x = \sin^{-1} 0.3674$$

$$= 22^\circ$$

(iii) $AC^2 = a^2 + b^2 - 2ab \sin C$

when $a = 10$, $b = 16$ *and* $C = (180 - 36 - 22) = 122$

$$= 10^2 + 16^2 - 2 \times 10 \times 16 \times \cos 122^\circ$$

$$= 525.57$$

$$AC = \sqrt{525.57}$$

$$= 23 \text{ km}$$

M6 – APPLICATIONS OF TRIGONOMETRY

ANSWER SET B

1 *Using Pythagoras Theorem*

$x^2 = a^2 + b^2$

when $a = 150$ *and* $b = 70$

$$= 150^2 + 70^2$$

$$= 27400$$

$$x = \sqrt{27400}$$

$$= 165 \text{ km}$$

2 *Using the sine ratio*

$$\sin \theta = \frac{O}{H}$$

$$\sin 12^\circ 45' = \frac{12}{x}$$

$$x = \frac{12}{\sin 12^\circ 45'}$$

$$= 54.4 \text{ cm}$$

3 *(i)* $\angle FAB = (360 - 345) + 75$

$$= 90^\circ$$

(ii) $75 + 74 = 149^\circ T$

(iii) Using Pythagoras Theorem

$$BF^2 = 40^2 + 52^2$$

$$= 4034$$

$$BF = \sqrt{4034}$$

$$= 65.6$$

$$= 66 \text{ m}$$

4 $\tan 19^\circ = \frac{83}{x}$

$$x = 241 \text{ m}$$

5 *(i)* $\angle ABC = 45 + (180 - 159)$

$$= 66^\circ$$

(ii) $AC^2 = a^2 + b^2 - 2ab \sin C$

when $a = 55$, $b = 73$ *and* $C = 66$

$$= 55^2 + 73^2 - 2 \times 55 \times 73 \times \sin 66$$

$$= 5087.9$$

$$AC = \sqrt{5087.9}$$

$$= 71.3 \text{ km}$$

(iii) Using the sine rule

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

when $A = \angle ACB$, $a = 55$, $B = 66^\circ$ *and* $b = 71.3$

$$\frac{\sin A}{55} = \frac{\sin 66}{71.3}$$

$$\sin A = \frac{55 \sin 66}{71.3}$$

$$A = \sin^{-1} 0.7047$$

$$= 44.8052$$

$$= 45^\circ$$

M7 – SPHERICAL GEOMETRY

ANSWER SET A

1 $l = \frac{\theta}{360} \times 2 \pi r^2$

when $\theta = 120$ *and* $r = 12$

$$= \frac{120}{360} \times 2 \times \pi \times 12^2$$

$$= 25.13 \text{ cm}$$

2 *(i)* $151^\circ - 131^\circ = 20^\circ$

(ii) $4 \times 20^\circ = 80$

$$= 1 hr\ 20 mins$$

3 *(i)* $32^\circ - 8^\circ = 24^\circ$

(ii) $60 \times 24^\circ = 1440 \text{ Nm}$

(iii) 22 knots is 22 Nm per hour. In 36 hours they travel

$36 \times 22 = 792 \text{ Nm}$

No. They will not be able to deliver on time as they can only travel 792 km in 36 hours.

4 *(i)* $130^\circ + 38^\circ = 168^\circ$

(ii) $l = \frac{\theta}{360} \times 2 \pi r^2$

$$= \frac{168}{360} \times 2 \times \pi \times 6400^2$$

$$= 18765.78 \text{ km}$$

(iii) $4 \times 168^\circ = 672 \text{ mins}$

$$= 11 \text{ hrs } 12 \text{ mins}$$

5 *(i)* $74^\circ + 12^\circ = 86^\circ$

(ii) $86 \times 4 = 344 \text{ minutes} = 5 \text{ hrs } 44 \text{ minutes}$

M7 – SPHERICAL GEOMETRY

ANSWER SET B

1 $l = \frac{\theta}{360} \times 2\pi r^2$

$$= \frac{360 - 70}{360} \times 2 \times \pi \times 5^2$$

$$= 25.3 \text{ m}$$

2 *(i)* $135^\circ - 77^\circ = 58^\circ$

$4 \text{ mins} \times 58^\circ = 232 \text{ mins}$

$$= 3 hrs\ 25 \text{ mins}$$

(ii) Later as Osaka is East of New Dehli

(iii) 12 PM – 3*hrs* 52*mins* = 8.08 AM

3 *(i)* $56^\circ + 1^\circ = 57^\circ$

$57^\circ \times 60 = 3420\ M$

(ii) $time = \frac{distance}{speed}$

$= \frac{3420}{425}$

$\approx 8 hrs$

(iii) 5PM *Monday* + 8*hrs* = 1 AM *Tuesday*

4 $55^\circ + 42^\circ = 97^\circ$

$l = \frac{0}{360} \times 2\pi r^2$

$= \frac{97}{360} \times 2 \times \pi \times 6400^2$

$= 10835\ km$

5 *Convert hours to minutes*

*3 hrs 44 mins *60 = 244 mins*

When each degree equals 4 mins

$\theta = \frac{minutes}{4}$

$\theta = \frac{224}{4}$

$= 56^\circ$

$98^\circ W - 56^\circ = 42^\circ W$

The coordinates of Town B are $(10^\circ S\ 42^\circ W)$

PB3 – MULTISTAGE EVENTS

ANSWER SET A

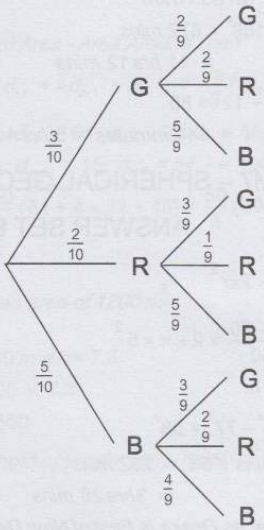
1 *Heads Heads* *Tails Tails*
Heads Tails *Tails Heads*

2 *(i)* $P(6) = \frac{1}{6}$

(ii) $P(not\ 6) = 1 - \frac{1}{6} = \frac{5}{6}$

(iii) $P(even) = \frac{3}{6} = \frac{1}{2}$

3 *(i)*



(ii) $P(Both\ Blue) = \frac{5}{10} \times \frac{4}{9} = \frac{2}{9}$

(iii) $P(1\ Red\ 1\ Green) = \left(\frac{2}{19} \times \frac{3}{9}\right) + \left(\frac{3}{10} \times \frac{2}{9}\right) = \frac{2}{15}$

(iv) $P(At\ Least\ One\ Red) = 1 - P(Not\ Red)$

$= \left(\frac{8}{10} \times \frac{7}{9}\right) = \frac{17}{45}$

4 $2 \times 5 \times 3 = 30$

Samantha has a combination of 30 3 course meals that she can choose from

5 *(i)* $3 \times 2 \times 1 = 6$

(ii) $P(Second\ Card\ N) = \frac{1}{3}$

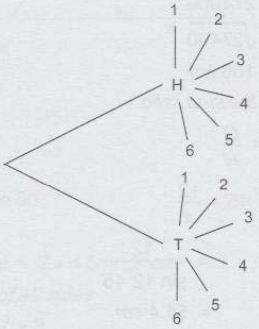
(iii) $P(First\ Card\ Not\ O) = \frac{2}{3}$

PB3 – MULTISTAGE EVENTS

ANSWER SET B

1 *TTT* *TTF* *TFT* *TFF*
FTT *FTF* *FFT* *FFF*

2 *(i)*



By either calculation or counting arms on probability tree

(i) $P(Head\ and\ Even) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

(ii) $P(\ Head\ or\ 6\ ) = \frac{1}{12}$

(iii) $P(\ Tails\ and\ not\ 1\ ) = \frac{5}{12}$

3 *(i)* $P(Win\ 1st\ Prize) = \frac{4}{100} = 0.04$

(ii) $P(Win\ 2nd\ Prize\ only) = \frac{96}{100} \times \frac{4}{99} \times \frac{95}{98} = \frac{304}{8085}$

$= 0.0376$

(iii) $P(Not\ Win) = \frac{96}{100} \times \frac{95}{99} \times \frac{94}{98} = \frac{7144}{8085} = 0.8836$

4 $5 \times 4 \times 3 \times 2 \times 1 = 120$

There are 120 ways 5 people can be seated.

5 *(i)* $P(1st\ Card\ is\ a\ Hearts) = \frac{1}{4}$

(ii) $P(Both\ Cards\ Hearts) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

6 $11 \times 10 = 110$

There are 110 different ways to choose a captain and vice captain.